5.4 Kinetic theory

Monatomic gas

Pressure	$p=\frac{1}{3}nm\langle c^2\rangle$	(5.77)	p pressure; n number density $=N/V$; m particle mass; $\langle c^2\rangle$ mean squared particle velocity
Equation of state of an ideal gas	$pV=NkT$	(5.78)	V volume; k Boltzmann constant; N number of particles; T temperature
Internal energy	$U=\frac{3}{2}NkT=\frac{N}{2}m\langle c^2\rangle$	(5.79)	U internal energy
Heat capacities	$C_V=\frac{3}{2}Nk$	(5.80)	C_V heat capacity, constant V
	$C_p=C_V+Nk=\frac{5}{2}Nk$	(5.81)	C_p heat capacity, constant p
	$\gamma=\frac{C_p}{C_V}=\frac{5}{3}$	(5.82)	γ ratio of heat capacities
Entropy (Sackur–Tetrode equation)[a]	$S=Nk\ln\left[\left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2}\mathrm{e}^{5/2}\frac{V}{N}\right]$	(5.83)	S entropy; $\hbar$ $=$(Planck constant)$/(2\pi)$; e $=2.71828\ldots$

[a]For the uncondensed gas. The factor $\left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2}$ is the quantum concentration of the particles, n_{Q}. Their thermal de Broglie wavelength, λ_T, approximately equals $n_{\mathrm{Q}}^{-1/3}$.

Maxwell–Boltzmann distribution[a]

Particle speed distribution	$\mathrm{pr}(c)\,\mathrm{d}c=\left(\frac{m}{2\pi kT}\right)^{3/2}\exp\left(\frac{-mc^2}{2kT}\right)4\pi c^2\,\mathrm{d}c$	(5.84)	pr probability density; m particle mass; k Boltzmann constant; T temperature; c particle speed
Particle energy distribution	$\mathrm{pr}(E)\,\mathrm{d}E=\frac{2E^{1/2}}{\pi^{1/2}(kT)^{3/2}}\exp\left(\frac{-E}{kT}\right)\mathrm{d}E$	(5.85)	E particle kinetic energy $(=mc^2/2)$
Mean speed	$\langle c\rangle=\left(\frac{8kT}{\pi m}\right)^{1/2}$	(5.86)	$\langle c\rangle$ mean speed
rms speed	$c_{\mathrm{rms}}=\left(\frac{3kT}{m}\right)^{1/2}=\left(\frac{3\pi}{8}\right)^{1/2}\langle c\rangle$	(5.87)	c_{rms} root mean squared speed
Most probable speed	$\hat{c}=\left(\frac{2kT}{m}\right)^{1/2}=\left(\frac{\pi}{4}\right)^{1/2}\langle c\rangle$	(5.88)	$\hat{c}$ most probable speed

[a]Probability density functions normalised so that $\int_0^\infty \mathrm{pr}(x)\,\mathrm{d}x=1$.

Transport properties

Mean free path[a]	$l = \frac{1}{\sqrt{2}\pi d^2 n}$	(5.89)	l d n	mean free path molecular diameter particle number density
Survival equation[b]	$\mathrm{pr}(x) = \exp(-x/l)$	(5.90)	pr x	probability linear distance
Flux through a plane[c]	$J = \frac{1}{4} n \langle c \rangle$	(5.91)	J $\langle c \rangle$	molecular flux mean molecular speed
Self-diffusion (Fick's law of diffusion)[d]	$\boldsymbol{J} = -D \nabla n$	(5.92)		
	where $D \simeq \frac{2}{3} l \langle c \rangle$	(5.93)	D	diffusion coefficient
Thermal conductivity[d]	$\boldsymbol{H} = -\lambda \nabla T$	(5.94)	$\boldsymbol{H}$ λ	heat flux per unit area thermal conductivity
	$\nabla^2 T = \frac{1}{D} \frac{\partial T}{\partial t}$	(5.95)	T ρ	temperature density
	for monatomic gas $\lambda \simeq \frac{5}{4} \rho l \langle c \rangle c_V$	(5.96)	c_V	specific heat capacity, V constant
Viscosity[d]	$\eta \simeq \frac{1}{2} \rho l \langle c \rangle$	(5.97)	η x	dynamic viscosity displacement of sphere in x direction after time t
Brownian motion (of a sphere)	$\langle x^2 \rangle = \frac{kTt}{3\pi\eta a}$	(5.98)	k t a	Boltzmann constant time interval sphere radius
Free molecular flow (Knudsen flow)[e]	$\frac{\mathrm{d}M}{\mathrm{d}t} = \frac{4R_\mathrm{p}^3}{3L} \left(\frac{2\pi m}{k} \right)^{1/2} \left(\frac{p_1}{T_1^{1/2}} - \frac{p_2}{T_2^{1/2}} \right)$	(5.99)	$\frac{\mathrm{d}M}{\mathrm{d}t}$ R_p L m p	mass flow rate pipe radius pipe length particle mass pressure

[a]For a perfect gas of hard, spherical particles with a Maxwell–Boltzmann speed distribution.
[b]Probability of travelling distance x without a collision.
[c]From the side where the number density is n, assuming an isotropic velocity distribution. Also known as "collision number."
[d]Simplistic kinetic theory yields numerical coefficients of 1/3 for D, λ and η.
[e]Through a pipe from end 1 to end 2, assuming $R_\mathrm{p} \ll l$ (i.e., at very low pressure).

Gas equipartition

Classical equipartition[a]	$E_\mathrm{q} = \frac{1}{2} kT$	(5.100)	E_q k T	energy per quadratic degree of freedom Boltzmann constant temperature
Ideal gas heat capacities	$C_V = \frac{1}{2} f N k = \frac{1}{2} f n R$	(5.101)	C_V C_p	heat capacity, V constant heat capacity, p constant
	$C_p = Nk \left(1 + \frac{f}{2} \right)$	(5.102)	N f n	number of molecules number of degrees of freedom number of moles
	$\gamma = \frac{C_p}{C_V} = 1 + \frac{2}{f}$	(5.103)	R γ	molar gas constant ratio of heat capacities

[a]System in thermal equilibrium at temperature T.